

SOML MEET 2
EVENT 3
Number Sense

NAME: _____
TEAM: _____
SCHOOL: _____

1. [2 Points] Simplify and give the exact value of $2 \cdot 2^{2000} + 3 \cdot 2^{2001}$. Express your answer as a single exponential expression (a number raised to a power).

ANS: _____

2. [3 Points] If you multiply the first 2002 prime numbers, how many zeros will there be at the end of the product?

ANS: _____

3. [5 Points] By definition, $n! = 1 \cdot 2 \cdot 3 \cdot 4 \cdot \dots \cdot n$. If $A = 20! \cdot 19! \cdot 18! \cdot 2! \cdot 1!$, then how many times does 3 appear in the prime factorization of A?

ANS: _____

SOML MEET 2
EVENT 3
Number Sense

NAME: Key
TEAM: _____
SCHOOL: _____

1. [2 Points] Simplify and give the exact value of $2 \cdot 2^{2000} + 3 \cdot 2^{2001}$. Express your answer as a single exponential expression (a number raised to a power).

Solution:

$$2 \cdot 2^{2000} + 3 \cdot 2^{2001} = 2^{2000} (2 + 3 \cdot 2) = 2^{2000} \cdot 8 = 2^{2000} \cdot 2^3 = 2^{2003}$$

ANS: 2²⁰⁰³

2. [3 Points] If you multiply the first 2002 prime numbers, how many zeros will there be at the end of the product?

Solution:

Each time a 2 and a 5 can be paired up the factor of 10 results. Each factor of 10 produces a zero at the end of the product. Since the product is of prime numbers, there is only one way to get a 10, from 2 and 5. So, there will be only 1 zero at the end of the product.

ANS: 1 zero

3. [5 Points] By definition, $n! = 1 \cdot 2 \cdot 3 \cdot 4 \cdot \dots \cdot n$. If $A = 20! \cdot 19! \cdot 18! \cdot 2! \cdot 1!$, then how many times does 3 appear in the prime factorization of A?

Solution:

In the expression $20! \cdot 19! \cdot 18! \cdot \dots \cdot 2! \cdot 1!$, the number 20 will appear as a factor once (in $20!$), 19 will appear twice (in $20!$ and $19!$), 18 will appear three 3 times (in $20!$, $19!$, and $18!$), and so on, until we get to 1, which will appear as a factor twenty times (in $20!$, $19!$, $18!$, ..., and $1!$). Only the 18's, 15's, 12's, 9's, 6's and 3's will "contribute" 3's to the prime factorization of A.

Each of the three 18's will have two 3's in its prime factorization, accounting for six 3's. Each of the six 15's will have one 3 in its prime factorization, accounting for six more 3's. Each of the nine 12's will have one 3 in its prime factorization, accounting for nine 3's. Each of the twelve 9's will have two 3's in its prime factorization, accounting for twenty-four 3's. Each of the fifteen 6's will have one 3 in its prime factorization, accounting for fifteen 3's. Each of the eighteen 3's will contribute one more 3 to the prime factorization of A. In all, there will be seventy-eight 3's in the prime factorization of A.

ANS: 78