

SOML MEET 1
EVENT 2
PROBLEM SOLVING

NAME: _____
TEAM: _____
SCHOOL: _____

1. [2 Points] Three people play a game in which one person loses and two people win each game. The loser must double the amount of money that each winner has at that time.

For example, if players A, B, and C have \$100, \$50, and \$10, respectively, and player A loses, Player A would have to give \$50 to player B and \$10 to player C. In this same situation, a loss for either player B or player C would put that person in debt.

Three players agree to play three games. Player A lost the first game, then B lost, then C lost. At the end of those three games, each player has \$46. What amount did player A start with?

A: _____

2. [3 Points] Mary wrote down all of the natural numbers from 1 to 300 inclusive. Jerry erased all of the multiples of 3. Larry erased all of the multiples of 4. How many numbers remained?

ANS: _____

3. [5 Points] Using all of the digits 1, 2, 3, 4, 5 and 6, it is possible to form 720 different six-digit numbers. If these are listed from least to greatest, what will be the 555th number on the list?

ANS: _____

**SOML MEET 1
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NAME: Key
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1. [2 Points] Three people play a game in which one person loses and two people win each game. The loser must double the amount of money that each winner has at that time.

For example, if players A, B, and C have \$100, \$50, and \$10, respectively, and player A loses, Player A would have to give \$50 to player B and \$10 to player C. In this same situation, a loss for either player B or player C would put that person in debt.

Three players agree to play three games. Player A lost the first game, then B lost, then C lost. At the end of those three games, each player has \$46. What amount did player A start with?

Solution: Assume that the players are called A, B, and C, and that they lose in that order. Work backwards:

	Player A's amount	Player B's amount	Player C's amount
after all 3 games	\$46.00	\$46.00	\$46.00
before player C lost	\$23.00	\$23.00	\$92.00
before player B lost	\$11.50	\$80.50	\$46.00
before player A lost	\$74.75	\$40.25	\$23.00

A: \$74.75

2. [3 Points] Mary wrote down all of the natural numbers from 1 to 300 inclusive. Jerry erased all of the multiples of 3. Larry erased all of the multiples of 4. How many numbers remained?

Solution 1: Imagine putting all of the 300 numbers into a Venn diagram with two overlapping circles, one (on the left) for the multiples of 3, the other (on the right) for the multiples of 4. The overlap of the two circles would contain the multiples of 12. 25 of the numbers that Mary wrote would belong there. 100 of the numbers that Mary wrote would belong in the circle on the left, but 25 have already been accounted for (in the overlap). Thus, 75 of the numbers would belong in the non-overlapping portion of the left circle. 75 of the numbers that Mary wrote belong in the circle on the right, but 25 have already been accounted for (in the overlap). Thus, 50 of the numbers would belong in the non-overlapping portion of the right circle. Thus, there would be a total of $75 + 25 + 50 = 150$ numbers in the three circles. These would be the 150 numbers erased, leaving another 150 numbers unerased.

Solution 2: Consider what happens to the first 12 numbers:

Mary writes some numbers:	1 2 3 4 5 6 7 8 9 10 11 12
Jerry erases multiples of 3:	1 2 x 4 5 x 7 8 x 10 11 x
Larry erases multiples of 4:	1 2 x x 5 x 7 x x 10 11 x

This same pattern will occur in every block of 12 numbers, half are erased and half remain. Therefore, 150 numbers left unerased.

ANS: 150

3. [5 Points] Using all of the digits 1, 2, 3, 4, 5 and 6, it is possible to form 720 different six-digit numbers. If these are listed from least to greatest, what will be the 555th number on the list?

Solution: 1/6 of the numbers on the list will start with 1, 1/6 of the numbers will start with 2, and so on. Thus, we can break the list into six blocks of 120 numbers: 120 numbers starting with 1, 120 numbers starting with 2, and so on. To get the 555th number, we must go through 4 of these blocks and partway through the 5th block. Thus, we are seeking the 75th number in the 5th block.

All of the numbers in the 5th block look like this: 5 _ _ _ _ . 1/5 of the numbers in this block will have a 1 in the second position, 1/5 will have a 2 in the second position, and so on (keeping in mind that the second position cannot be filled with a 5). Thus, we can break the block of 120 numbers into five smaller blocks of 24. To get the 75th number in this block, we must go through three of these smaller blocks and partway through the fourth. Thus, we are seeking the 3rd number in the 4th block.

So, we are seeking a number that begins with 54, in particular the 3rd smallest such number. Counting up, we see that the numbers go like this: 541236, 541263, 541326. Therefore, the 555th number on the original list will be 541326

ANS: 541326